

**QUASI-STATIC ANALYSIS  
OF A  
FOUR WHEELED VEHICLE  
DURING COMBINED MANEUVERS**

by

**Patrick Ryan Turner**

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Michigan Technological University  
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Signature of Author



Department of Mechanical Engineering

May 23, 1985

Certified by



J. Karl Hedrick  
Thesis Supervisor

Accepted by



Ain A. Sonin  
Chairman, Department Thesis Committee

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## **Abstract**

Combined maneuver analysis is used to predict vehicle response to simultaneous tractive effort and cornering or braking and cornering. In this thesis, a rubber tired vehicle has been modelled to provide a vehicle suspension design tool that considers a combined maneuver condition. Much effort has been given to making the model generally applicable in any stage of suspension design. The model is developed using force variables. This allows easier interaction with various suspension and tire models. In this way, spindle deflections are calculated off-line so specific suspension geometry and force distribution characteristics need not be considered in the main model or in early design iterations. The above concept is exercised on a simple vehicle. Suggestions are made for future modifications and use of a more comprehensive vehicle representations. The major strength of this work is the general applicability of the modelling approach, the full derivation of the system equations, and the simple use of the personal computer based program.

Thesis Supervisor: J. K. Hedrick  
Title: Professor of Mechanical Engineering

## Acknowledgements

I've often wondered just how informal this page could be? As an effort to express my true feelings about this research and my stay at MIT I regard this as "my page" and hope not to detract from the information in this document.

Studying at MIT is easily the hardest task I've ever undertaken, and starting a fresh project with little or no former training is a great strain. At times only strong friendships and a loving family is all that kept me going. I don't regard myself as an academic but rather one with a need to know. Sometimes the desire to know did not satisfy the requirement for knowledge at MIT. Never-the-less, this, the last formal requirement, will accomplish the seemingly impossible. There are many who deserve a thank-you much greater than words but...

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