

Integrating the Mechanism Optimal Synthesis Tool into the Mechanical Design Industry

by

Patrick Ryan Turner, P.E.
Engineering Manager of Mechanical Computer Aided Engineering

Schlumberger Technologies, CAD/CAM Division
Ann Arbor, Michigan 48106

August 4, 1989

Abstract

At AUTOFACT '88 (in SME paper MS88-711), a method and computer program were presented which combined optimization and synthesis to create a powerful mechanism design tool[2]. The Mechanism Optimal Synthesis Tool is a graphic oriented computer program which performs kinematic "dimensional synthesis" of both planar and spatial mechanisms. In this paper, case studies and discussion on how this tool has been integrated into the mechanical design environment will be presented. This will include an automotive suspension design example and an eight bar transport mechanism example. Discussed is how this approach makes possible the solution of design problems usually heavily dependent on trial and error and experience. This information will also be extrapolated to mechanical design in general.

1 INTRODUCTION

A trend toward design synthesis is beginning to emerge in the mechanical design community. In the advanced design groups of some companies there seems to be a move away from analysis oriented design to the synthesis of designs. Or, perhaps more accurately, there is a leveraging of analysis to promote the synthesis of designs. Design synthesis is being applied in several mechanical engineering disciplines. Structural design, shape design, mechanical linkage design are just a few. The process of synthesizing a design allows the engineer to focus more on the *specification* or *functionality* of a design. This, in place of a focus on the *process* involved in design or analysis (i.e. the task of making a geometric

model, performing a finite element analysis, etc.). So , synthesis allows the engineer to expend more energy directly on what makes the company money, *the product design*.

As with any emerging technology a starting point or test-bed for a synthesis product was needed. At Schlumberger Technologies, CAD/CAM Division the synthesis of general 2D and 3D Kinematic Machines was the starting point. The design of “linkages” or “mechanisms”, as they are called, lend themselves well to the study of design synthesis. This is primarily because, the function of a kinematic linkage is mostly dependent on the spatial relationship of the joints and not on the geometric shape of the individual parts. This makes automatically changing the analysis model easier by moving joint connection locations (markers) on individual parts.

The synthesis of any linkage design can be broken into two separate problems; namely *Type* and *Dimensional* synthesis. “Type synthesis” is the process of selecting the basic class of mechanism (e.g. 4 bar, 6 bar, planar, spatial, etc.) that exhibits some of the motion behavior required by a given application. Once the class or topology of mechanism is selected by Type Synthesis, it can be used as input to “Dimensional synthesis”. “Dimensional synthesis” is the process of refining the design to the specific motion requirements of the new application. Because the bulk of mechanism design is done by starting with a design used in a similar situation, “Dimensional synthesis” seemed a viable and more practical place to start.

In this paper the use of the Bravo Mechanism Optimal Synthesis Tool (BravoMOST¹) software system is described via two case studies. BravoMOST (or MOST as it will be referred to for the rest of this paper) is used to perform dimensional synthesis on a 3D automotive suspension and a 2D transport mechanism. MOST provides a means to explore the benefit to the engineer/designer of mechanism synthesis.

2 The Approach to Optimal Synthesis Used By MOST

MOST provides the ability to perform kinematic “dimensional synthesis” of mechanisms[1]. This means that the system will change the dimensions of the mechanism to satisfy some design objective. However, the system will not change the topology of the mechanism (a six bar Watt type mechanism will remain a six bar Watt type, etc.). The “dimensional synthesis” system for mechanisms is described below and seen in Figure 1.

¹ BravoMOST is a 2D/3D mechanisms synthesis package produced by Schlumberger Technologies, CAD/CAM Division, Ann Arbor, MI.

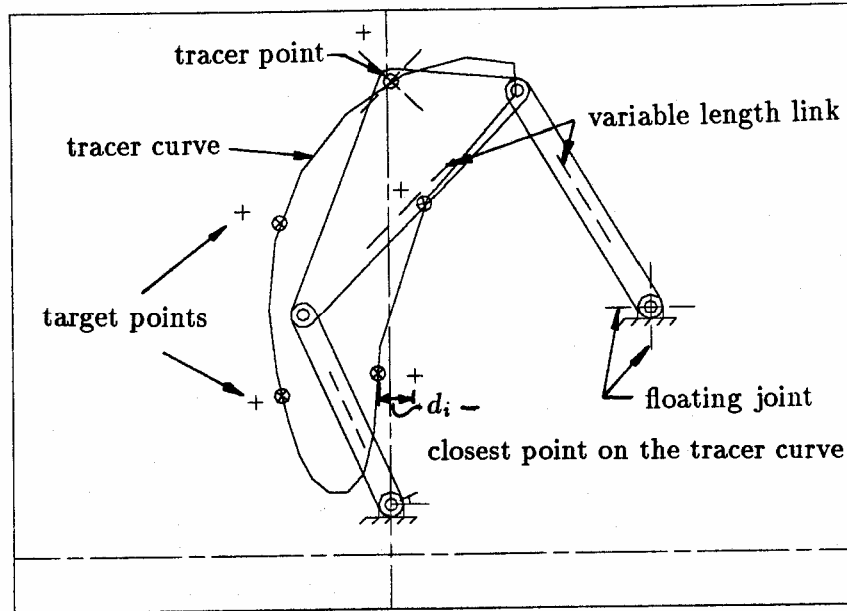


Figure 1.: MOST is shown being used on a standard 4bar mechanism.

A graphic display and user interface assist in describing the design problem. The starting mechanism is first read from a file. This file is in an industry standard (ADAMS²) format.

Next, the design intent is specified by defining a new path that the mechanism should trace. The new path is represented by a series of “target points” which defines a “path objective”. A “tracer point” is created (shown on the mechanism as the circled cross). This tracer point identifies the position on the mechanism which must pass through the target points.

The creation of “design variables” controls which dimensions MOST is allowed to modify in the design. “Design variables” (shown as the triple dashed lines) can be links with variable length, joint locations that can be moved around on a part, etc. These design variables are modified by the optimizer until the closest approximation of the new path is achieved.

An optimization algorithm is used to determine how the design variables must change in order for the tracer point to come closer to the “objective”, i.e. the desired new path. The optimization algorithm requires an analysis (in this case a kinematic analysis is used) to determine gradients (slopes) which are used to determine the best way to change the design variables. These changes are made to the mechanism (design variables) and the kinematic analysis is repeated to determine how closely the objective has been met.

² ADAMS (Automatic Dynamic Analysis of Mechanical Systems) is a 3D mechanisms analysis package produced by MDI (Mechanical Dynamics Inc., Ann Arbor, MI.)

The kinematic analysis described above is a proprietary package provided as an integral part of the system. The kinematic analysis is used to solve for the positions of the mechanism parts as the path is traced. Thereby providing visualization/animation of the assembled mechanism to the user as well as generating information needed by the optimization.

The initial mechanism itself may be defined using any means available to create an input file in the ADAMS format. This can range from a text editor to commercially available graphic mechanism modelers. In most cases a user of MOST would have existing designs in the ADAMS format that could be used as a starting point. Assuming the user of MOST would be likely to use some form of dynamic analysis program (ADAMS) in normal design analysis. ADAMS was deemed an appropriate choice because of its wide spread acceptance and its analysis capabilities.

The design variables and objectives are specified through a FIT³ (Flexible Interface Tool) graphic based interface. A graphic representation of the mechanism is displayed with full 3D view control. Interaction with the system is provided by “picking” the mechanism display graphics, making selections from menus, etc. The interface provides the advantages of programmability and customizing capabilities. The interface may be altered to conform to standard terminology for a specific industry, or, changed using the provided command language to make it more specific to a given design situation or user preference. The command language allows the creation of extensive macros or procedures to automate a specific command sequence or specify parametric operations.

3 Automotive Suspension Design

The first case study will describe the modification of an automobile suspension. An SLA (Short Long Arm) suspension shown in Figure 2 will be used. This suspension has a specific topology that remains constant during the synthesis problem. Meaning, the number of parts, joints, joint types used, and how they are connected together will not change. However, the dimensions of parts are modified so the SLA is optimized for the particular vehicle⁴. The SLA is most often used in the front of a vehicle because of the way it packages around the engine and its favorable performance and durability characteristics.

³ FIT (Flexible Interface Tool) is a product of Schlumberger Technologies, CAD/CAM Division, Ann Arbor, Michigan.

⁴ The different applications vary depending on vehicle size, handling requirements, ride requirements, packaging constraints, cost targets, etc.

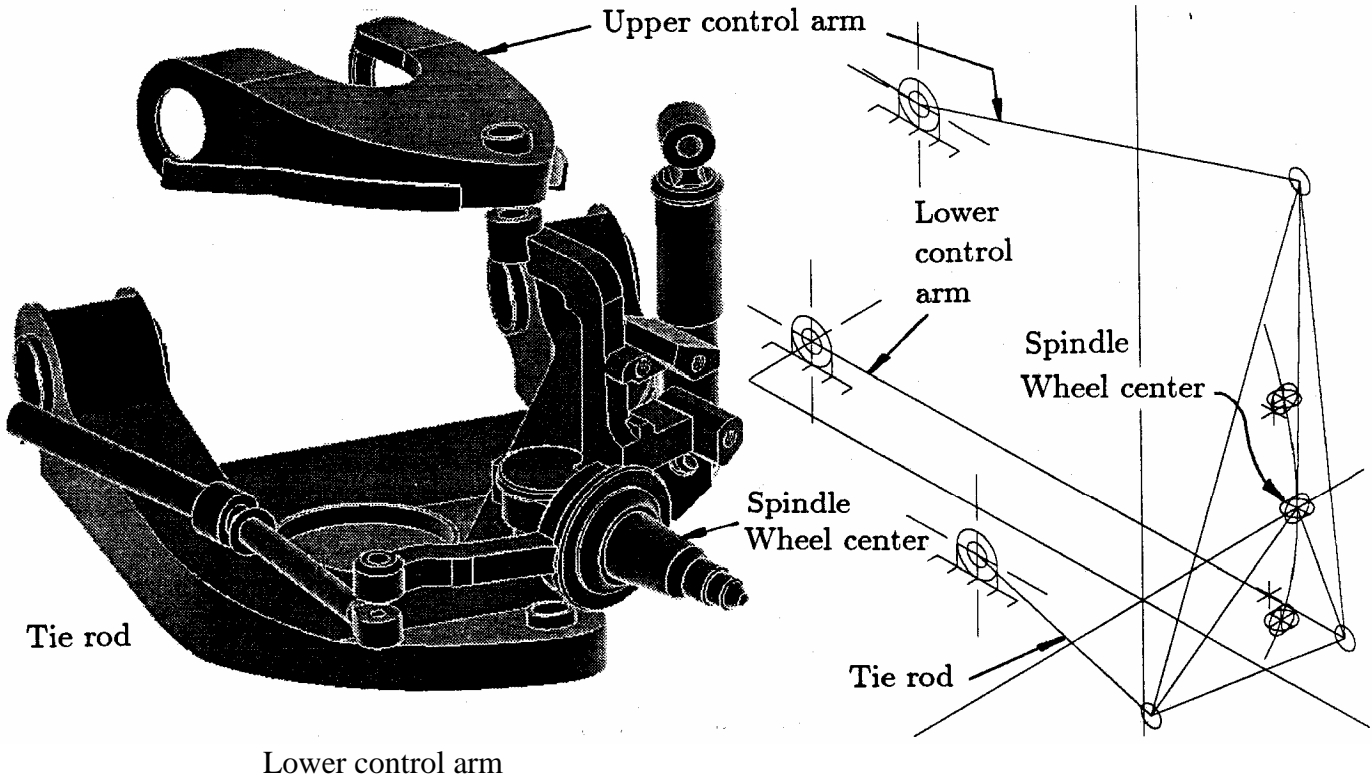
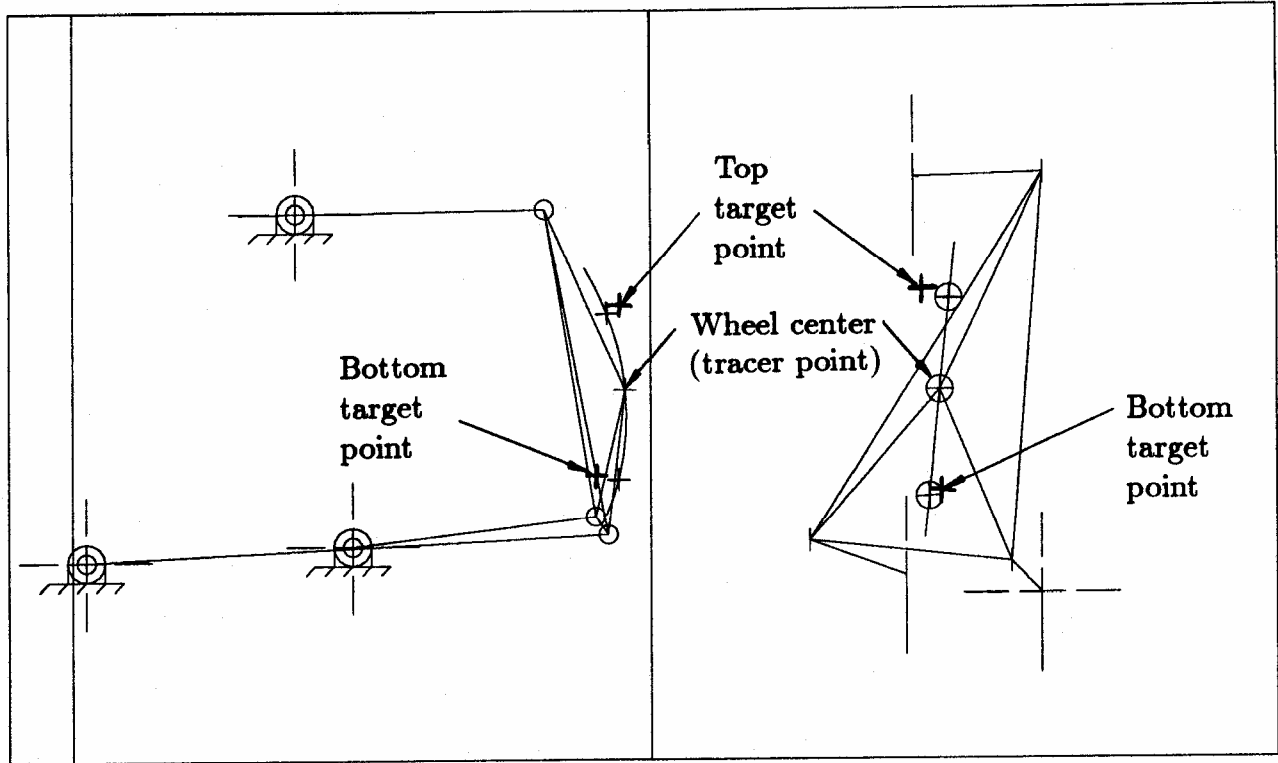


Figure 2: Solid model and MOST model of the SLA suspension.

The functional design of an SLA suspension is done considering the movement of the wheel center as it moves up and down. The wheel center's movement traces a three dimensional path. An engineer will evaluate this path in the front (camber), side (caster), and top (toe) views[3]. These camber, caster, toe curves, as they are called, are analyzed individually and relate directly to vehicle ride and handling. The reason automotive engineers consider these curves separately (with 2D computer models) is because these curves relate in non-linear ways and until now no adequate 3D design tool has existed. A change to one suspension dimension to effect, say, the camber curve may have an ill effect on the toe curve, etc. So, when doing manual design, it's much easier to optimize the curves individually. Even so, once each curve is optimized in 2D it must be combined with the other 2D designs into a real 3D suspension (maintaining all the 2D attributes) This is nearly impossible and therefore trade-offs are made. The result is a lengthy trial and error design process.

MOST would attempt to modify all three (camber, caster, and toe) curves simultaneously using optimal synthesis. Optimization is well suited to find an acceptable solution while considering non-linearities and a relatively large number of design parameters. So, computer algorithms replace the need for the engineer to balance all of these non-linearly related parameters.



Front (camber) view **Side (caster) view**
 Figure 3: MOST is shown being used on the SLA suspension mechanism.

Specifically for this case study, a change to the SLA suspension is specified by placing target points that redefine the camber and caster curve. From a global perspective the top of the camber curve was to be “tipped away from the vehicle” and the caster curve was to be rotated, top toward the front. The next sections describe how MOST was used to do this.

3.1 Synthesis Problem Setup

The general procedure for setting up the synthesis problem for the SLA suspension is described in section 2. The creation of a tracer point, creation of target points, and specification of design variables is shown in Figure 3. The tracer point is placed at the wheel center position. The general shape of the trace curve (path traced by the wheel center) is dictated by the suspension topology. Because no local changes were required in the middle of the trace curve, only two target points were used at either end of the curve. So, in this case the changes to the overall orientation of the trace curve were required. The selection of design variables varied during the synthesis, therefore, this portion of the problem set up is described in the next section. Next, some particular setup tactics for the SLA suspension will be described.

The SLA suspension is a two dof (degree-of-freedom) mechanism. The first dof corresponds to the vertical “bounce” motion of the suspension. This dof is controlled by a rotational generator on the pivot joint connecting the control arm to ground. The second dof corresponds to

the steering motion of the spindle. This dof is controlled by a translational generator on the tie rod joint. The MOST version used for this synthesis only allowed single dof models to be used. This did not pose a problem because the “steering dot” was “locked down” by replacing the tie rod joint with a connection to ground. Once this change was made, the SLA suspension was read into MOST from an ADAMS file.

For some mechanisms that are relatively large, system performance can be improved by altering some system tolerances. This is not required but will help response time in some cases. Since the scale of the SLA suspension is relatively large, described in millimeters (*mm*), the kinematic solver “assembly tolerance⁵” was increased. This sped up the kinematic solver assembly convergence with only an acceptably small degradation in the solution quality. Also, the optimization “minimum step size” was increased for a similar reason. The optimizer would avoid taking small steps that only decreased the objective function a very small amount, therefore, converge to the solution faster. It should be noted using the default setting for these parameters would have worked, just more slowly.

The MOST system will, by default, assume a single dof mechanism with a rotational motion generator can move the generator through 360°’s. For mechanisms like the SLA, this is not the case. Therefore, an operating range of the SLA mechanism was specified. Limiting the range of motion imparted by the motion generator can be specified in two ways: by entering values that specify a range, between which the motion generator should operate (e.g. -30° to 30° for the SLA) or, by selecting target points that indicate where the path should begin and end. The net result of either method is to limit the motion of the mechanism to only a portion of its possible range of motion. If desired the system can be allowed to find the range by letting it move the mechanism until it “locks up” or binds.

For the SLA suspension the range was first specified by generator value. Then later it was found that the generator range that was appropriate for the start of the synthesis problem was not accurate as the mechanism was changed by MOST. So, when this was discovered the range was redefined using the target points. By using the target points to define the boundaries of the operating range, MOST automatically updates the generator values in order to stay within the target point boundaries while the mechanism dimensions are changed.

3.2 Solution Tactics

The goal of the synthesis was to influence the camber and caster curves of the SLA suspension simultaneously, as described at the end of section 3. So, as previously discussed, the two target points were placed in 3D space to correspond with the position of each of the desired 2D curve end points simultaneously.

⁵ The distance a joint can be separated and still be considered connected for the purposes of assembling a mechanism.

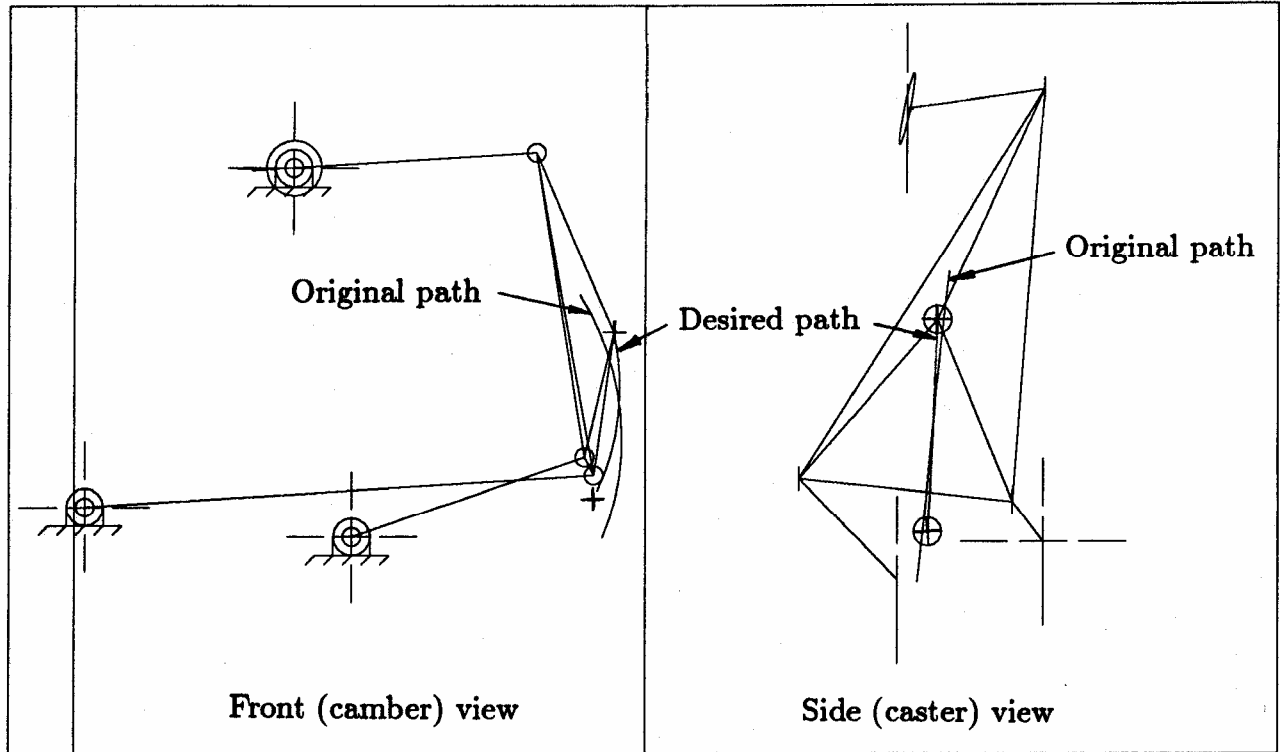


Figure 4: The final SLA suspension mechanism.

Selecting the most effective design variables for optimization can be a difficult task. This could have been especially true for the SLA suspension. However, MOST provided a means that made it much simpler. To start, design variables were selected based on the designers intuition. It was quickly determined that the SLA was very counter-intuitive. So, the system was reset to the initial state and “everything” was made a design variable. Then, a design sensitivity report was generated by the system. All design variables with small sensitivities (one or two orders of magnitude smaller than the largest) were deleted. This resulted in a 25% reduction in the number of design variables. Then, the design sensitivities were re-checked after every few optimization steps to make sure the proper design variables were being used. Using this tactic the problem converged nicely. The solution is shown in Figure 4.

To give a dimensional perspective on the solution, the SLA mechanism was modeled in *mm* and was several hundred *mm* in size. The objective function started at approximately 26.0mm and the synthesis was stopped when the objective function reached <2.0mm.

Once the final mechanism was obtained an updated ADAMS file was written out for input to ADAMS dynamic analysis program or a CAD system. Then, the engineer could do all the dynamic simulation necessary to further verify the new design.

3.3 SLA Suspension Synthesis Summary

The goal of the SLA synthesis problem was to modify the camber and caster curves. MOST was able to do this quickly while lending an intuitive feel to the critical dimensional parameters of the design. Before the introduction of this technology, this type of design problem could typically take several weeks. Using MOST this was reduced to 2 days.

A major benefit to applying MOST's optimal synthesis to the SLA suspension was that all curves were satisfied simultaneously. This eliminated the need to combine subsequent 2D compromises by trial and error. More subtly, MOST gives an "optimal" solution. So, the engineer may specify very optimistic target points such that it may be impossible for the suspension to go directly through the desired path. In this case, MOST will make the wheel center get as close to that desired path as possible. Thus, helping the engineer get to those required compromises much more quickly.

When poor design variables were selected (as an inexperienced suspension designer may do) the system still *slowly* started to converge to the solution. Using the design sensitivities allows a less experienced engineer to select a set of design variables that are effective. This tactic may not have been necessary if a more experienced suspension designer was performing the design synthesis.

Once the problem setup and solution tactics were outlined for this initial problem the process could be automated. The problem setup and some of the solution procedures could be programmed in a command file. The rest of the "interactive tactics" could be outlined and a less experienced person could operate the system and get satisfactory results.

4 Transport Mechanism Design

The second case is the design of an eight bar transport mechanism which could be used for several applications. In this particular case it was to be used as a "film carrier". The film carrier is required to move in a high speed linear fashion at the top of the trace curve. This requires a smooth and continuous motion as seen in Figure 5. Also, curvilinear translation (no rotation) of the carrier part is required.

The difficult part in synthesizing this type of mechanism is that it has a tendency to "branch". Branching is a problem where the kinematic assembly solution of the mechanism suddenly jumps (branches) to a different way to assemble the mechanism[1]. This happens because there may be several valid solutions to the set of nonlinear equations that model the mechanism. These several valid solutions represent multiple ways to put the parts of the mechanism together. This type of branching is usually impossible for physical mechanisms because it requires tearing down and reassembly.

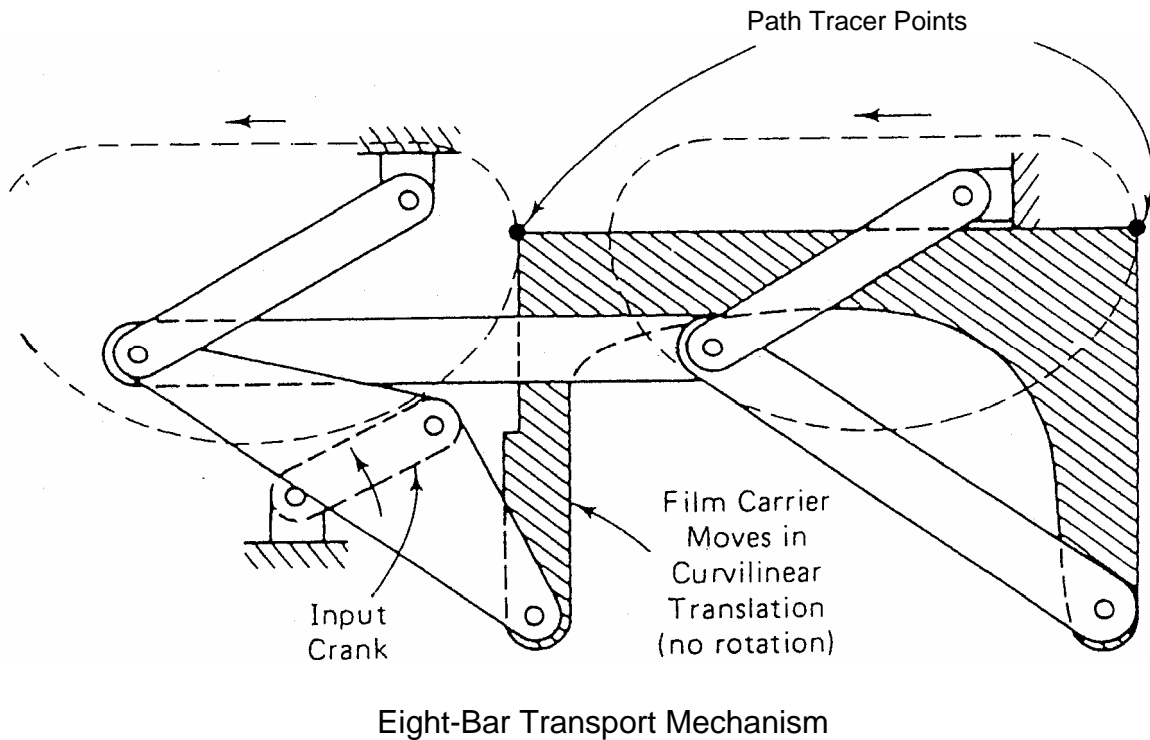


Figure 5: The desired path for the transport mechanism[4].

4.1 Synthesis Problem Setup

The initial setup of this problem was very simple. All system defaults were used except, a faster 2D kinematic solver was specified since this is a planar mechanism⁶. The rest of the problem setup follows that described in section 2 starting with reading in the ADAMS file.

The starting target points were selected to correspond to the critical straight-line portion of the desired path as seen in Figure 6. For the initial synthesis problem almost all possible candidates for design variables were used. As in the SLA suspension case, a design sensitivity analysis was used to determine the most effective set of design variables. This tactic was used throughout the design problem to assure the best set of design variable were being used.

4.2 Solution Tactics

It was quickly discovered that even a minor change to the sub-four bar mechanism shown in Figure 7 would cause locking of the transport mechanism. Therefore, the design variables corresponding to this sub-mechanism were deleted. For the rest of the design problem the dimensions of these parts remained fixed.

⁶ Both the 2D and 3D solvers come with the MOST system.

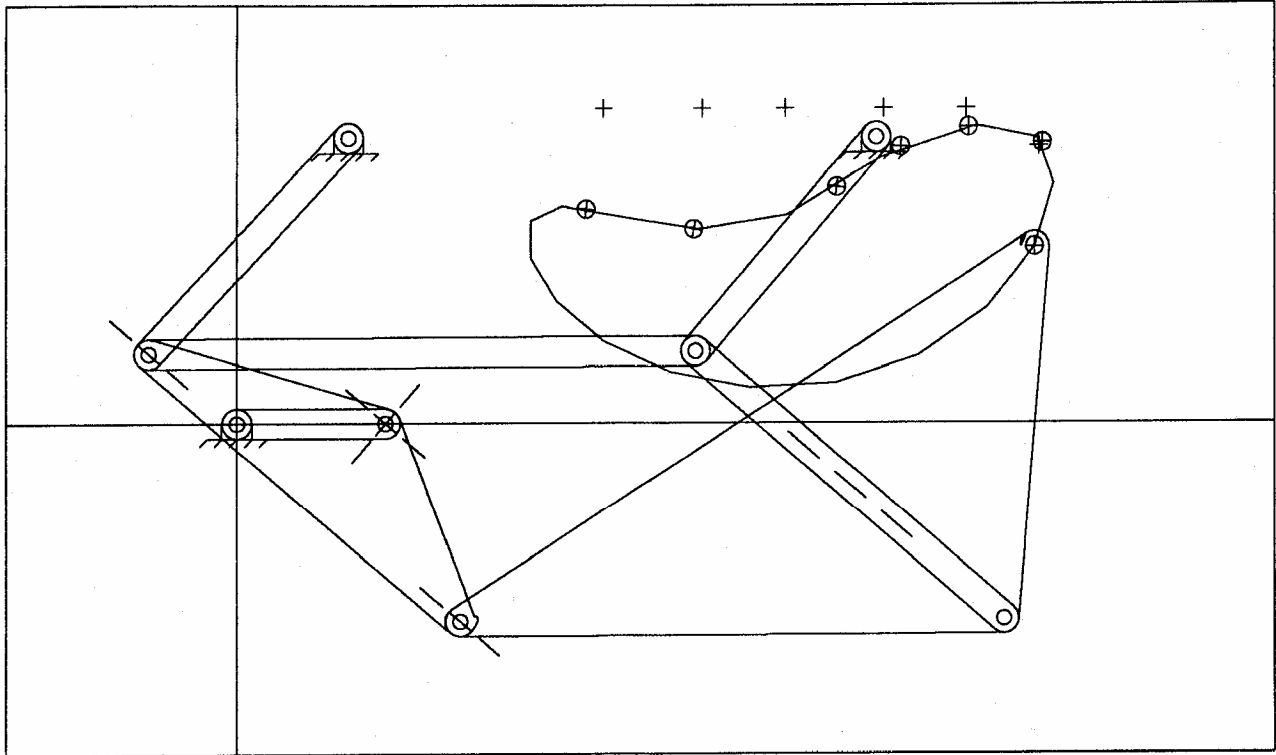


Figure 6: Initial setup of the transport mechanism.

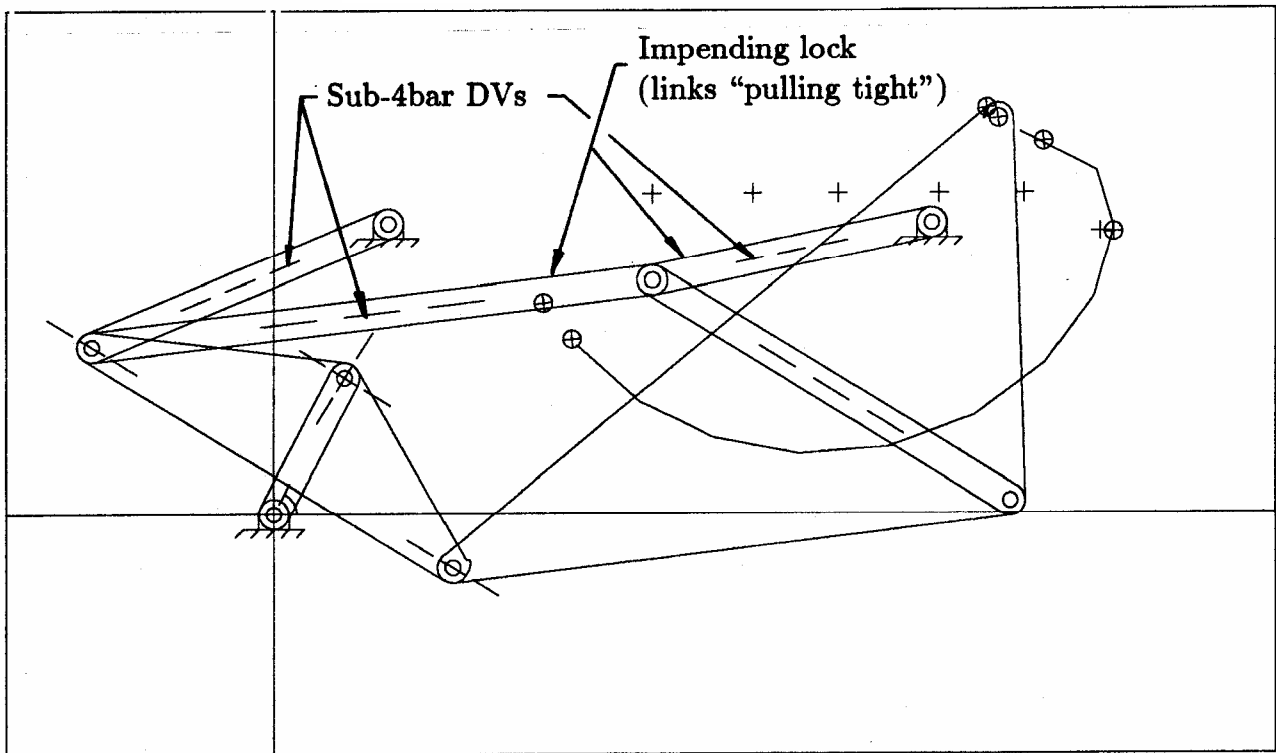


Figure 7: Minor changes would cause the sub-4bar to lock.

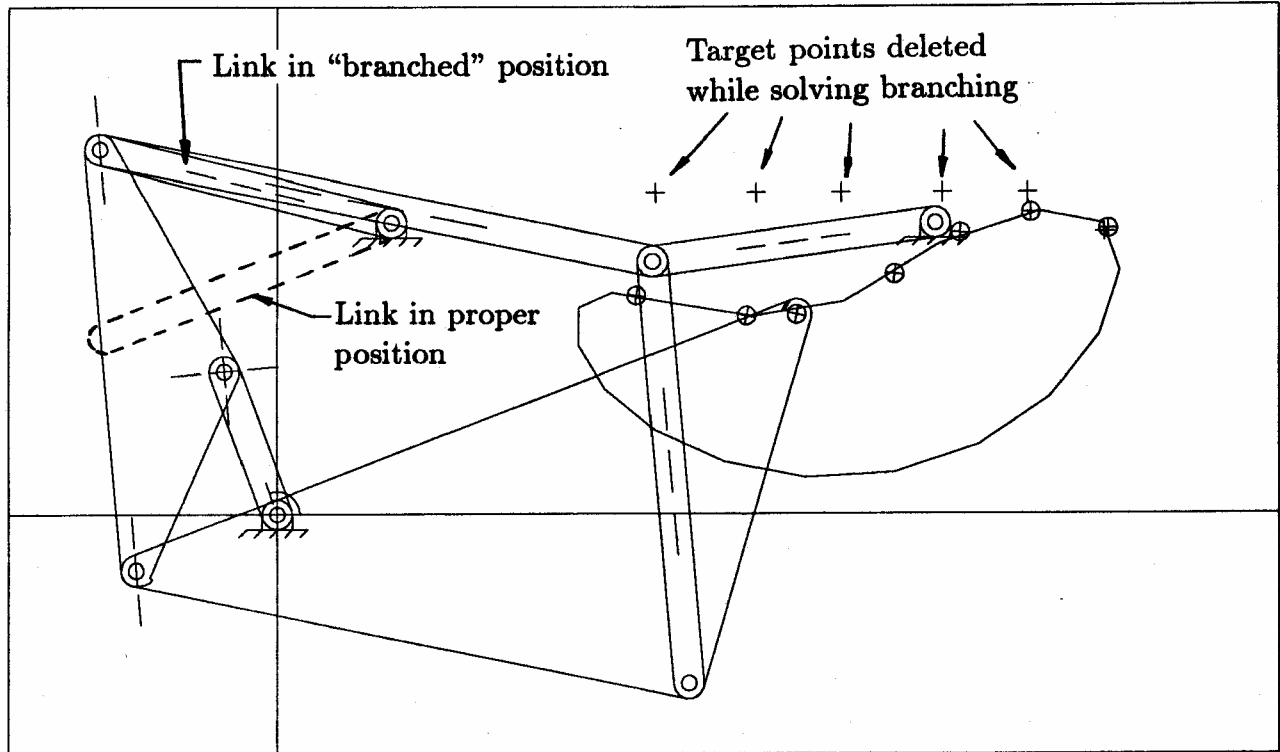


Figure 8: The transport mechanism would cause the kinematic solver to branch in the configuration shown.

Next, it was discovered that this mechanism would branch at the position shown in Figure 8. This may have gone un-noticed, but was easily detected when the engineer viewed the animated motion. When this was found, all initial DV's (design variables) and target points were deleted. The tracer point was relocated to the part causing the branching, one target point added, and one design variable defined as shown in Figure 9a. Then, by taking a few optimization steps the branching problem went away. So, even though the kinematic solver had branching problems⁷ with this mechanism, MOST was used to change the mechanism to make the branching go away. So, even though critics say optimization coupled with iterative analysis would not be practical, branching is a solvable problem in optimal synthesis. Next, the initial design problem was resumed.

The straight-line target points and DVs were re-added and design sensitivity rechecked to assure the proper DVs were being used (considering what had been learned from the "locking" and "branching" problems).

⁷ Iterative kinematic analysis is used to solve for the assembled positions of the parts of the mechanism. This type of iterative solution of non-linear equations in subject to convergence to multiple solutions, called branching.

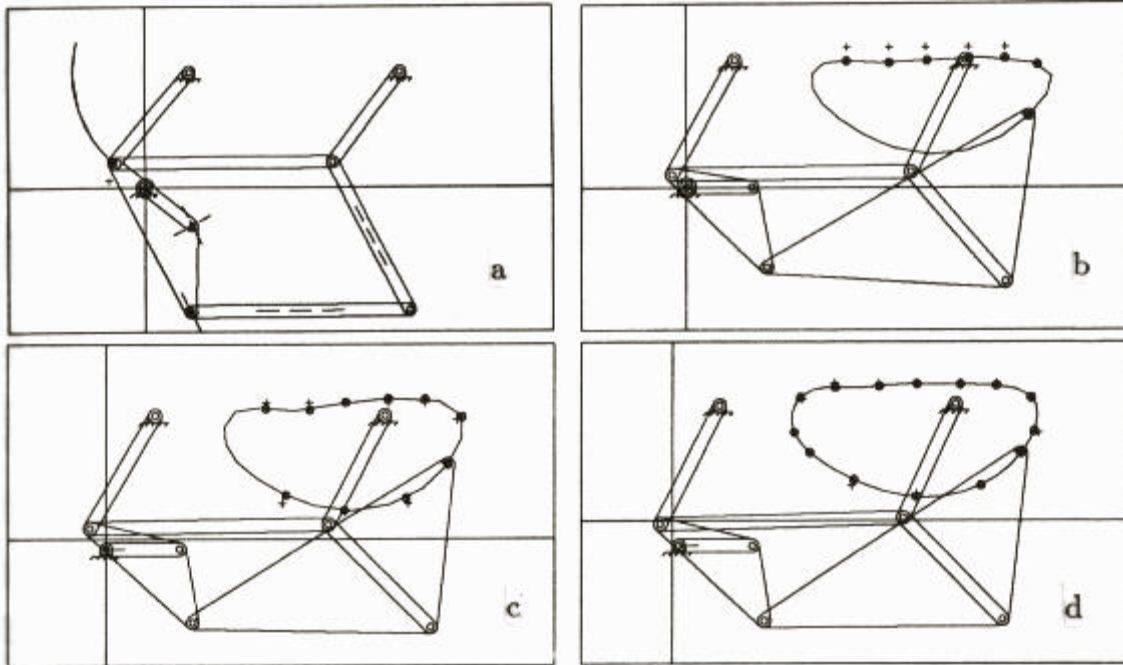


Figure 9: After branching was fixed as shown in (a) target points were repeatedly added (b,c, and d) to “mold” the trace curve.

After several optimization steps were taken the straight-line target points were satisfied. However, other portions of the trace curve became undesirable because of the need for a smooth continuous motion. So, additional target points were added to “mold” the trace curve as problems areas appeared. This tactic was repeated until the entire trace curve conformed to the desired path, and seen in Figure 9b,c and d.

During the solution of the transport mechanism, several DVs were identified as the most important to the solution of the design problem. By monitoring the changes to these DVs and performing sensitivity analyses, a feel for the design problem was obtained. These “important DVs” are shown in Figure 10.

During the entire MOST session, the transport mechanism would exhibit “non-assembly errors”. These errors indicated the kinematic solvers inability to assemble the mechanism at a particular configuration. For the transport mechanism these configuration happened near particular target points, during the optimization. In all cases the system would continue with the optimization step and still decrease the objective function (converge toward the solution). However, the kinematic solver tries very hard to *always* assemble the mechanism before it gives up. This would cause the optimization step to take a long time. So, to eliminate these frustrating but non-fatal assembly errors, the following was done. MOST was asked to assemble the mechanism as near the offending configuration as possible (using the set generator function). Then, the mechanism was visually inspected to see what links were “pulled tight”. If any of the

“tight” links had DVs on them, they were deleted. This eliminated the assembly errors during the optimization and significantly sped up the steps.

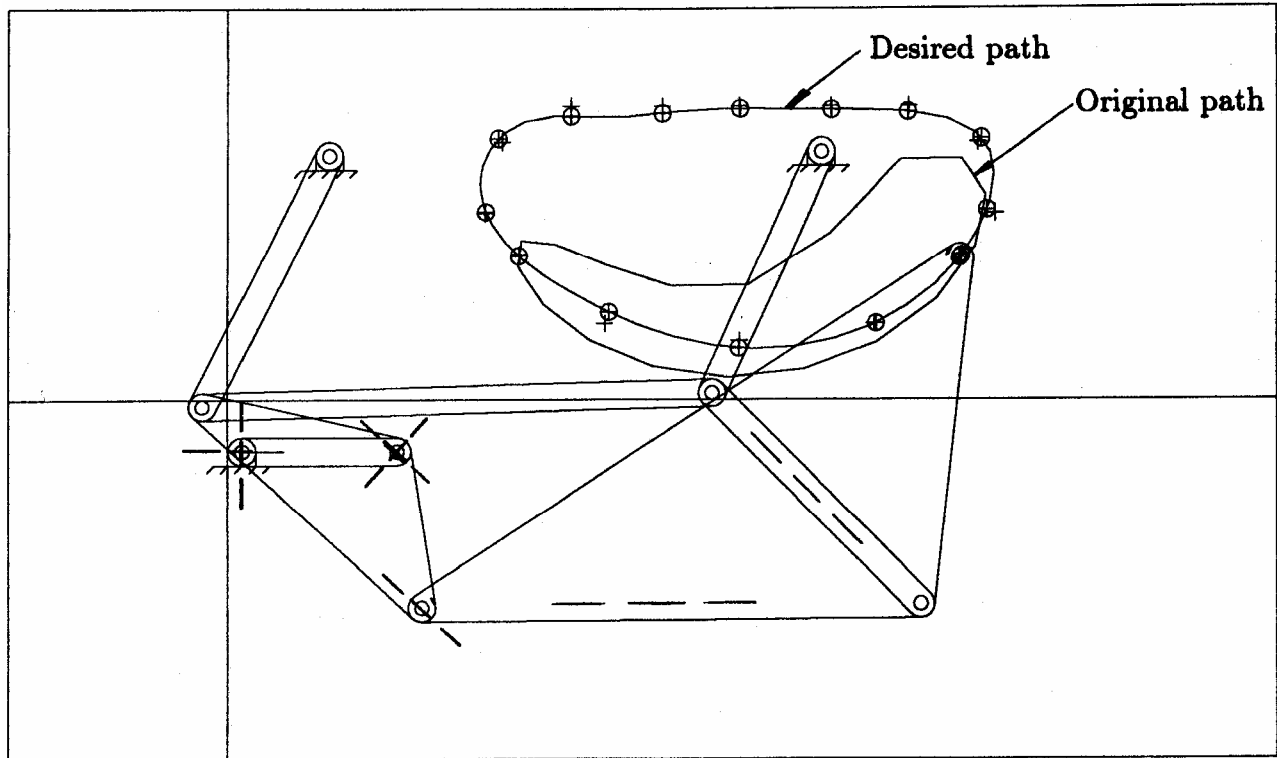


Figure 10: Several DVs were identified as the most important to the solution of the design problem. Giving a “feel” for the design solution space.

Once the final mechanism was obtained an updated ADAMS file was written out for input to ADAMS dynamic analysis program or a CAD system.

4.3 Transport Mechanism Design Synthesis Summary

Starting with an ADAMS file this problem was completed in approximately two hours. This demonstrates the ability to solve highly non-linear mechanism designs with relatively little prior knowledge about the design. The interactive molding of the trace curve by adding target points allowed the engineer to only deal with the desired path. Thus leaving the non-intuitive changes to the mechanism to the system.

The MOST system was used in a highly interactive manner allowing “what if” experimentation. This lead to a much deeper understanding of the design parameters, or solution space, in a short time. Features like setting the current DV value, generation of a design sensitivity report, and moving target points teaches the engineer about the mechanism. For example, identifying critical or sensitive part dimensions could provide tolerance information for manufacturing and assembly operations.

After the initial solution, this problem was retried from the beginning several times. Even small changes to the solution tactics used lead to widely varying solution paths. For each try the solution was ultimately reached with varying amounts of effort required. Therefore, the ability to repeat a particular solution path is low. This is likely due to the impact changes in tactics have to the “solution space” during the course of an interactive session. A slight change would cause the optimization algorithm to follow the design variable gradients in a different manner. Thus forcing the use of subsequently different solution tactics.

Supporting features like: animation, set configuration, visibility options, etc. allows the engineer to visually inspect the mechanism very closely at any time in the design process. This was particularly useful when dealing with non-assemble and branching. Just being able to identify that these problems exist was extremely valuable. Problems like this usually coincide with areas of low mechanical advantage and weak transmission angles. Things usually avoided in mechanism design.

5 Conclusions Provided by the Case Studies

The MOST optimal synthesis tool for mechanism design can solve real engineering design problems with little simplification. For example, there was no need to break up the SLA suspension into a series of idealized four bar mechanisms. There was no need to solve the eight bar transport mechanism in a piecewise fashion. This is new in the mechanism design area.

MOST was used successfully as an interactive design tool. This was questionable because of the iterative nature of the optimization and analysis algorithms. But the engineer could comfortably sit with the system and work on the design. This aside, major productivity gains were shown for both the suspension design and transport mechanism design problems. Taking problems usually requiring weeks to solve and doing them in hours or days.

The successful solution of the problems weighed heavily on the ability to control what the system could, and maybe more importantly, could not change during the optimization. Adding, deleting, and moving target points and adding, deleting, and setting the current value of DVs allowed this flexibility. Being able to “reset the design” to a previous state makes “what *if*” experimentation painless and backtracking to change solution strategies, when progress is slowed, easy.

Also accountable for the success of the MOST system were the AT-like (Artificial Intelligence) features included. These features allowed the system to compensate for some numerical ambiguities and still progress toward a solution. For example, when the kinematic solver had non-assembly problems, the system would ignore any potentially bogus information and proceed. Or, if the optimization would fail to decrease the objective function, the system would

intelligently change system parameters and retry. These types of things are only possible because of the system's *knowledge* about the type of problem being solved.

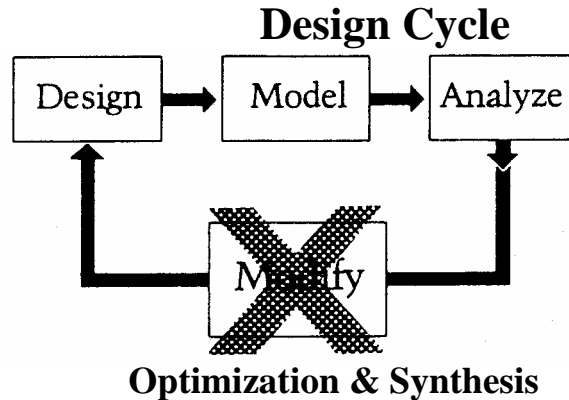


Figure 11: The design cycle is show with optimization and synthesis replacing the modify portion. This is the tactic used by MOST.

5.1 Integrating Into the Mechanical Design Environment

In general MOST improves engineering productivity by being targeted at the *modify* portion of the design cycle as shown in Figure 11. MOST can be well integrated into the mechanical design environment because in addition to performing dimensional synthesis of mechanisms it can teach the engineer about the mechanism and helps synthesize designs more quickly. Providing design sensitivity analysis is only one way MOST does this. The system is flexible enough to address a broad range of real problems seen in industry today. This applicability is enhanced because MOST solves spatial problems and is integrated with CAD and analysis via the ADAMS formatted input/output file.

Because MOST provides an optimal solution, experimentation with other topologies is more practical. This means that non-traditional mechanisms can be tried regardless if that type or class of mechanism is not generally used. This is because MOST will get the mechanism to function as near to the desired path as it can. So, experimentation is now possible that there was no time for in the past.

5.2 Eliminating Trial and Error and Required Experience Level

MOST has helped eliminate trial and error tactics in two ways. The first way by providing the ability to design/synthesize in 3D rather than combining the results from several 2D analyses. Because the 2D results combine non-linearly the engineer rarely gets the 3D result that is required. So, many iterations of the whole process are required in addition to the time spent figuring out how to idealize the different projections of the 3D design into several 2D

mechanisms. The second way is by automatically performing the necessary non-intuitive changes to the mechanism to get the desired path. This is especially true for 3D mechanism designs where selecting the proper changes to design parameters is difficult.

MOST has also provided a means to lessen the experience needed to successfully modify a mechanism for a new application. This is because the user interacts with the design on a high level and not the individual design parameters. This is especially true once a given problem type is setup. The programmability of MOST allows the duplication of solution tactics on different problems automatically. Then, a much less experienced person can solve subsequent problems using the automated strategy. This may be difficult if convergence through the “solution space” is sensitive to slight changes in tactics.

Finally, the system promotes “playing” with the mechanism to see what it can do. Thereby capitalizing on creativity. Analysis is not an end in itself!

6 Extrapolation of Optimal Synthesis to Mechanical Design in General

MOST has shown us that, as with any emerging technology, engineers must learn to apply the tool to the design problem. Not throw the design problem at the tool. This is because, (as with finite element analysis, for example) at the early stages of development, optimal synthesis technology won't contain all the features needed to solve all problems. However, great advantages in design cycle productivity are to be gained by proper application of the technology; like with FEA.

Optimal synthesis can make an engineer more productive by leveraging on creativity. This is done by removing the tedious task involved in manually iterating the design/analysis model and recursively repeating an analysis.

We must move toward letting the engineer interact with the design at a strategically high level, and not get bogged down by the process. Before synthesis technology can be wide spread, strong ties between the analysis model and the geometric model must be forged; much stronger than exists today. In fact the engineer should see no difference between the two. The creation of a design model should implicitly provide the analysis model. Only then will we see the power of design synthesis routinely being used in the drafting room and manufacturing shops.

7 ACKNOWLEDGEMENTS

The author would like to acknowledge the effort of the following colleagues. Jeff Cooper of the CAD/CAM division provided most of the information regarding the specific case studies

described and also provided valuable editorial comments. The efforts of Richard Stobby, of the CAD/CAM division, have also made this paper possible.

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